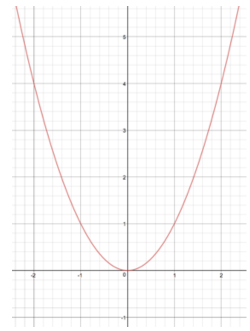
Math 251 Week 8A Activity – Section 4.1 Applications of the Derivative and Section 4.2 What Derivatives tell us

Section 4.1 Applications of the Derivative

Definition: Absolute Maximum and Minimum
Let *f* be defined on an interval *I* containing *c*.

- If $f(c) \geq f(x)$ for every *x* in *I*, then *f* has an *absolute maximum* value on *f*(*c*) on *I* at *c*.
- If $f(c) \leq f(x)$ for every *x* in *I*, then *f* has an *absolute minimum* value on *f*(*c*) on *I* at *c*.

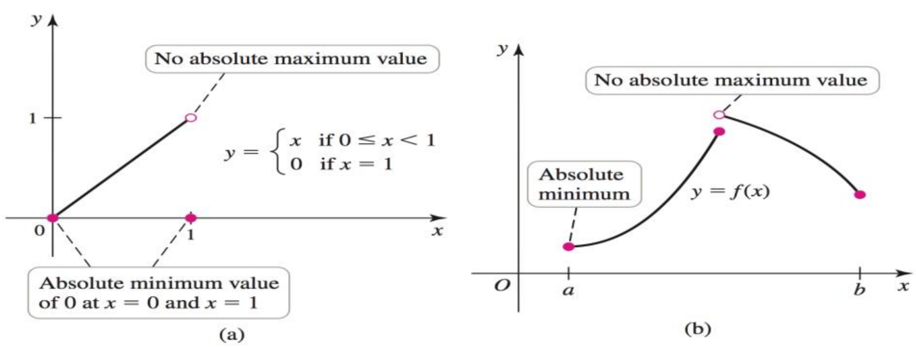
1. The function to the right, $f(x) = x^2$, has an absolute minimum at *x* = 0, but it does not have an absolute maximum. Can you explain why it doesn't have an absolute maximum?



2. Can you think of a function that has both an absolute maximum and an absolute minimum on an interval?

There are two conditions for the existence of an absolute minimum or absolute maximum – the function must be continuous on an interval and the interval must be closed and bounded.
Extreme Value Theorem: A function that is continuous on a *closed interval* [*a, b*] has an absolute maximum value and an absolute minimum value on that interval.

- For the functions in figures a and b, there is not an absolute maximum because the function is discontinuous at *x* = 1.



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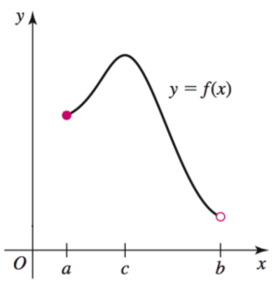
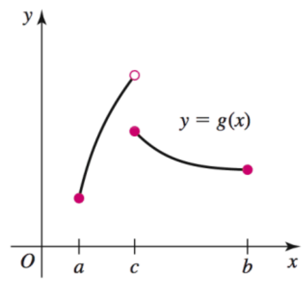
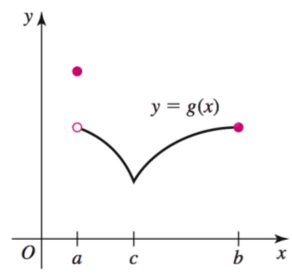
3. Can you sketch the graph of a function that is continuous on an interval but does not have an absolute minimum value?

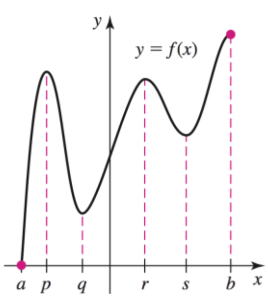
4. Determine the values at which the functions have an absolute maximum or an absolute minimum. Is there a point on the functions where the derivative is undefined?

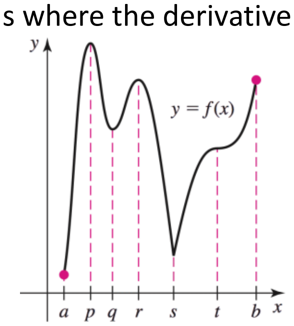


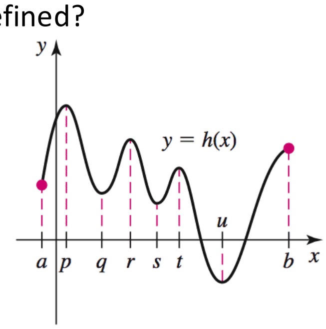




5. Determine the values on [a, b] at which the functions have a local and/or extreme value. Are there points on the functions where the derivative is undefined?







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Theorem 4.2 states that if a function *f* has a local minimum or maximum at *c* and $f'(c)$ exists, then $f'(c) = 0$.

5. If $f'(c) = 0$ for a function f, will the function always have a local minimum or maximum at c? Sketch examples to justify your answer.

Finding critical points

6. Find the critical points for the function on the given interval and determine whether the critical points correspond to a local minimum or local maximum or neither.

a. $f(x) = 12x^5 - 20x^3$ on [-2, 2]

b. $f(x) = \sin(x) \cdot \cos(x)$ on [0, 2π]

c. $f(x) = \frac{x}{(x^2+3)^2}$ on [-2, 2]

d. $f(x) = x^{2/3}$ on [-8, 8]

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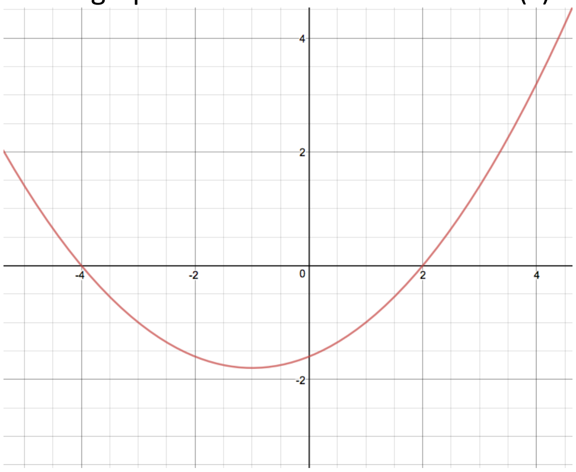
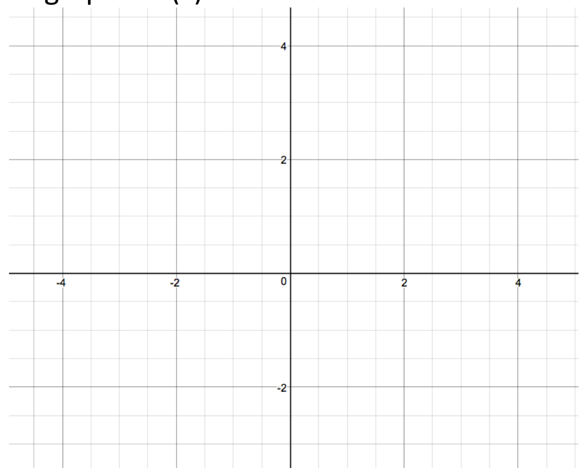
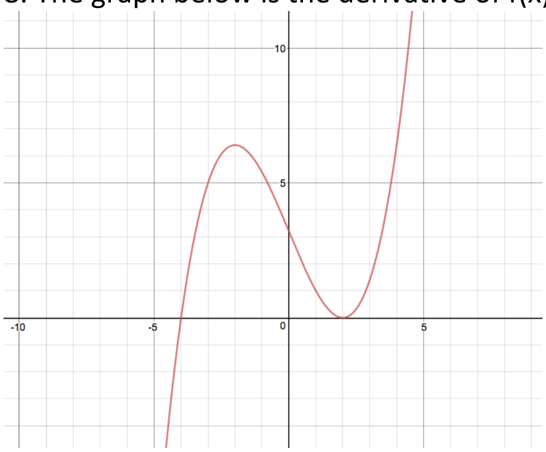
Section 4.2 What Derivatives Tell Us

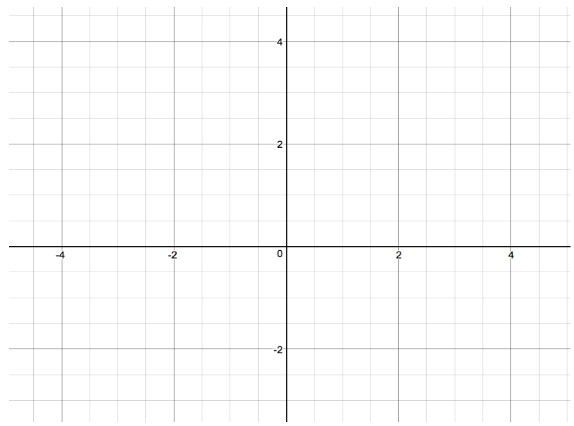
7. The graph below is the derivative of f(x). Sketch the graph of f(x) on the blank axes.





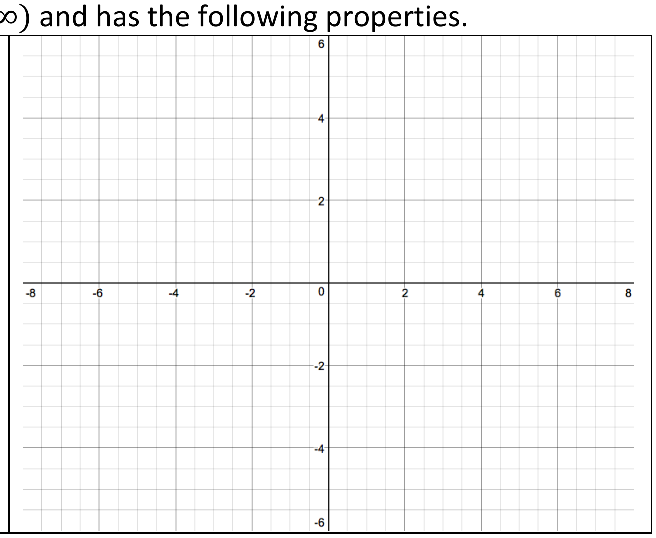
8. The graph below is the derivative of f(x). Sketch the graph of f(x) on the blank axes.





9. Sketch a graph that is continuous on $(-\infty, \infty)$ and has the following properties.

$f'(x) < 0$ on $(-\infty, 2)$
$f'(x) > 0$ on $(2, 5)$
$f'(x) < 0$ on $(5, \infty)$



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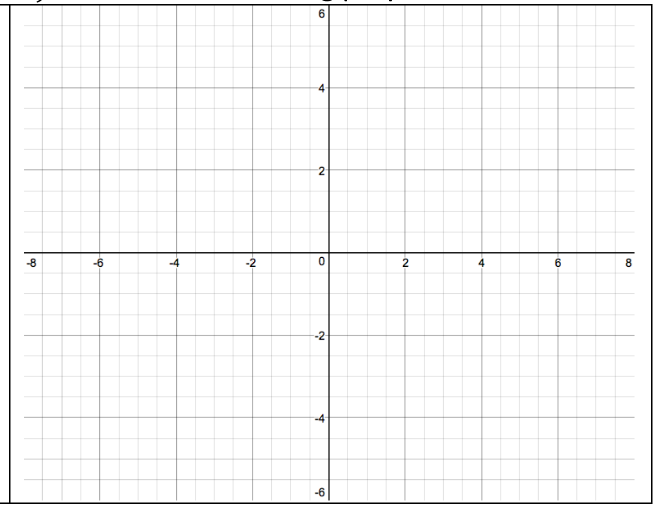
10. Sketch a graph that is continuous on $(-\infty, \infty)$ and has the following properties.

$f'(-1)$ is undefined
$f'(x) > 0$ on $(-\infty, -1)$
$f'(x) < 0$ on $(-1, \infty)$



11. Sketch a graph that is continuous on $(-\infty, \infty)$ and has the following properties.

$f(0) = f(4) = 0$
$f'(0) = f'(2) = f'(4) = 0$
$f(x) \geq 0$ on $(-\infty, \infty)$