Section 3.1 Definition of the Derivative and
Section 3.2 Working with Derivatives

Section 3.1 Definition of the Derivative

1. For a given function f, what does f' represent.

f' represents the slope at every point x in the domain of f.

2. Given a function f and a point a in its domain, what does $f'(a)$ represent?

$f'(a)$ represents the slope of the line tangent to $f(x)$ at the point a.

3. Give three different notations for the derivative of f with respect to x.

$f'(x)$ $\quad \partial_x f(x)$

$\frac{d}{dx} f(x)$

4. Evaluate the derivative of $f(x)$ at $x = a$ for the given value of a, **using the definition of derivative**. Then write an equation for the tangent line to $f(x)$ at $x = a$.

a. $f(x) = (x+1)^2$, $a = 2$

$f'(a) = \lim_{x \to a} \frac{(x+1)^2 - (a+1)^2}{x-a} = \lim_{x \to a} \frac{x^2+2x-a^2-2a}{x-a} = \lim_{x \to a} \frac{(x-a)(x+a)+2(x-a)}{x-a}$

$= \lim_{x \to a} (x+a)+2 = 2a+2.$ $\quad f'(2) = 6.$

$y - 9 = 6(x-2)$

b. $f(x) = \sqrt{x+1}$, $a = 0$

$f'(a) = \lim_{x \to a} \frac{\sqrt{x+1} - \sqrt{a+1}}{x-a} = \lim_{x \to a} \frac{x+1-a-1}{(x-a)(\sqrt{x+1}+\sqrt{a+1})} = \lim_{x \to a} \frac{x-a}{(x-a)(\sqrt{x+1}+\sqrt{a+1})}$

$= \lim_{x \to a} \frac{1}{(\sqrt{x+1}+\sqrt{a+1})} = \frac{1}{2\sqrt{a+1}}$; $f'(0) = \frac{1}{2}$

$y - 1 = \frac{1}{2}(x)$

c. $f(x) = \frac{1}{x+1}$, $a = 1$

$f'(a) = \lim_{x \to a} \frac{\frac{1}{x+1} - \frac{1}{a+1}}{x-a} = \lim_{x \to a} \frac{a-x}{(x-a)(x+1)(a+1)} = \lim_{x \to a} \frac{-1}{(x+1)(a+1)}$

$f'(a) = \frac{-1}{(a+1)^2}$; $f'(1) = -1/4.$ $\quad y - 1/2 = -\frac{1}{4}(x-1)$

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5. The graph of the function $f(x) = 1 - x^2$ is given below. Find the equations of the tangent lines at:

$f'(x) = -2x$

a. $x = -1$

$f'(-1) = 2 \quad f(-1) = 0$

$y = 2(x+1)$

b. $x = 0$

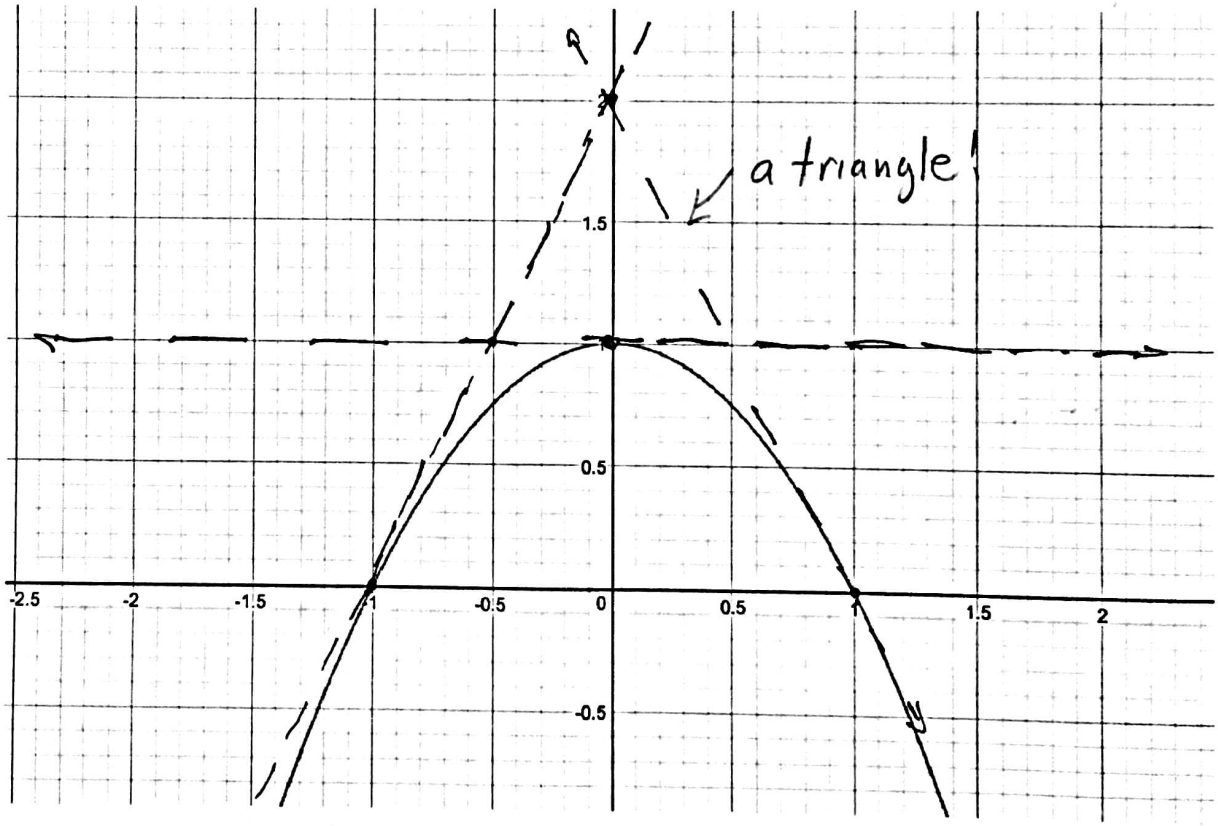
$f'(0) = 0$ $\quad y - 1 = 0$

$f(0) = 1$

c. $x = 1$ $\quad y = -2(x-1)$

$f'(1) = -2 \quad f(1) = 0$

Graph the tangent lines for $f(x)$ on the graph of $f(x)$. Explain what you notice about the tangent lines.



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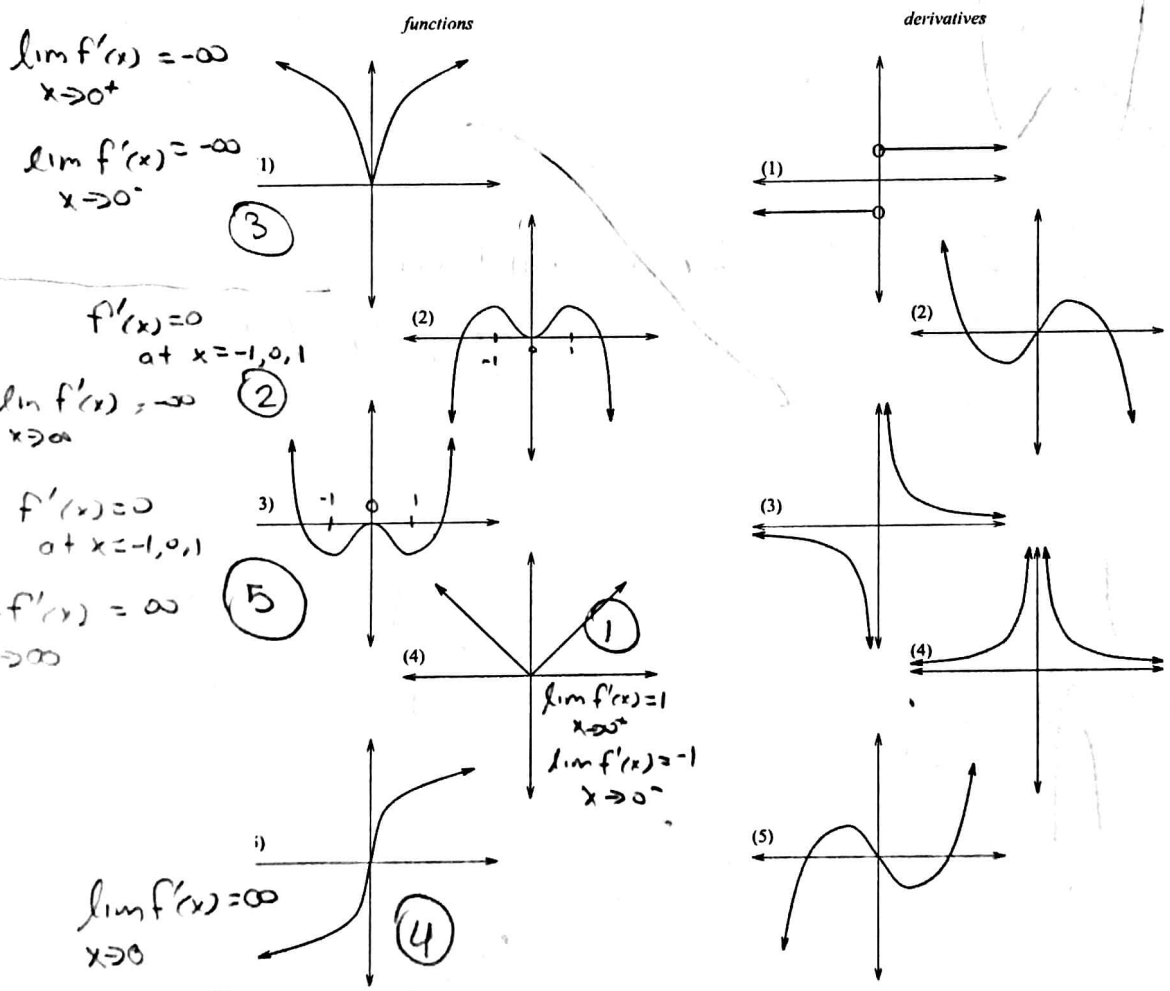
6. Match each graph of a function $f(x)$ with one of the graphs of a functions that is a derivative, $f'(x)$. Justify each matching.



$\lim_{x \to 0^+} f'(x) = -\infty$

$\lim_{x \to 0^-} f'(x) = -\infty$

3

$f'(x) = 0$ at $x = -1, 0, 1$

$\lim_{x \to \infty} f'(x) = -\infty$

2

$f'(x) = 0$ at $x = -1, 0, 1$

$\lim_{x \to \infty} f'(x) = \infty$

5

1

$\lim_{x \to \infty^+} f'(x) = 1$

$\lim_{x \to 0^-} f'(x) = -1$

$\lim_{x \to 0} f'(x) = \infty$

4

from left and right

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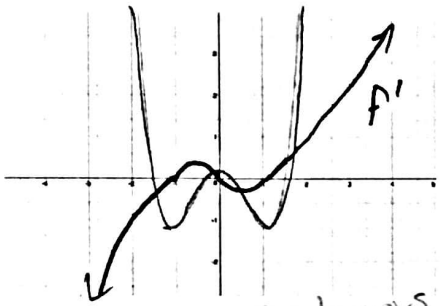
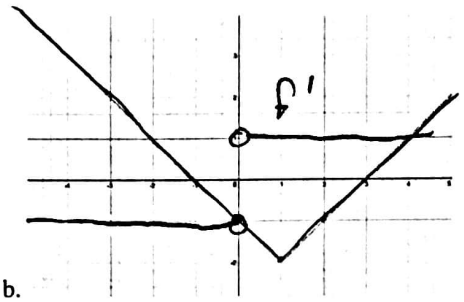
7. Sketch the graph of the derivatives of the following functions. Determine if the derivatives are continuous functions. Explain your answers.

a.



continuous, yes. Polynomial function, no holes or asymptotes.

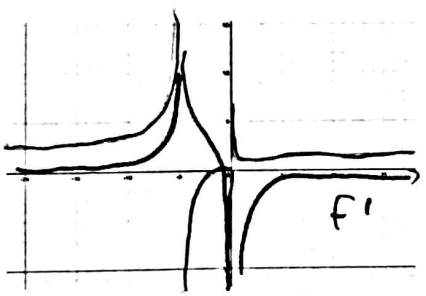
b.



not continuous at $x=0$

$\lim_{x\to 0^+} f'(x) \neq \lim_{x\to 0^-} f'(x)$

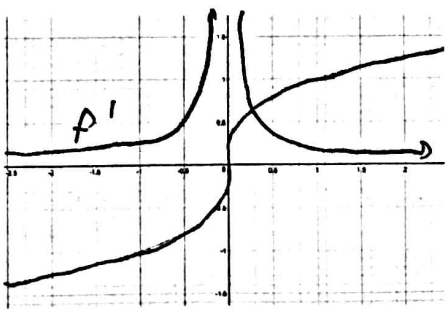
c.



no, $\lim_{x\to -5} f'(x) = \infty$

not continuous. $\lim_{x\to 0} f'(x) = -\infty$

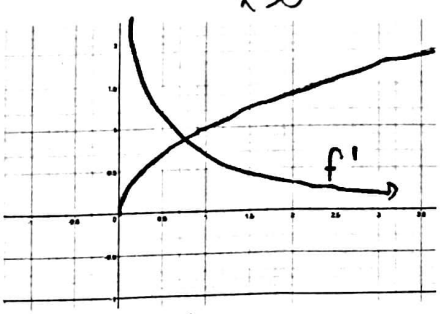
d.



not continuous

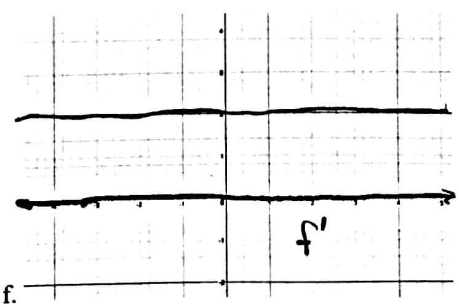
$\lim_{x\to 0} f'(x) = \infty$

e.



f' is continuous on it's domain $f': (0,\infty)$

f.



yes. $f'(x)$ is constant. continuous. No holes.

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8. Determine if the following are possible or not possible. Explain your answer and provide a graph to back up your explanation.

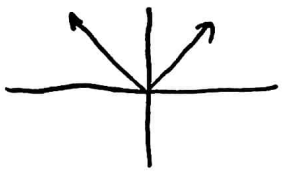
a. A function $f(x)$ is differentiable at a point x, but not continuous at that point.

not possible

Theorem: If f is differentiable at x then f is also continuous at x

b. A function $f(x)$ is not differentiable at a point x, but is continuous at that point.

yes



$f(x) = |x|$ is continuous but not differentiable at $x=0$

9. Graph the function $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x+1 & \text{if } x > 0 \end{cases}$

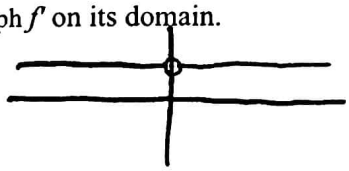
a. For $x < 0$, what is $f'(x)$?

$f'(x) = 1$, $x < 0$

b. For $x > 0$, what is $f'(x)$?

$f'(x) = 1$ $x > 0$

c. Graph f' on its domain.



d. Is f differentiable at 0? Explain.

it is not,

f is not continuous at 0, so it cannot be differentiable