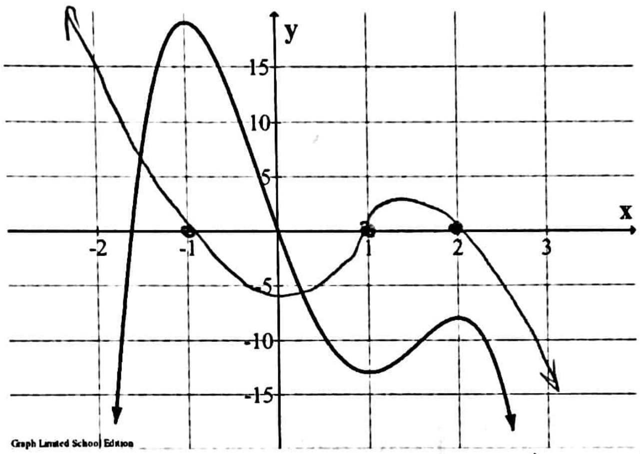
Week 4 Activity = Section 3.2 and 3.3 Review

Math 251 Derivatives involving products and quotients

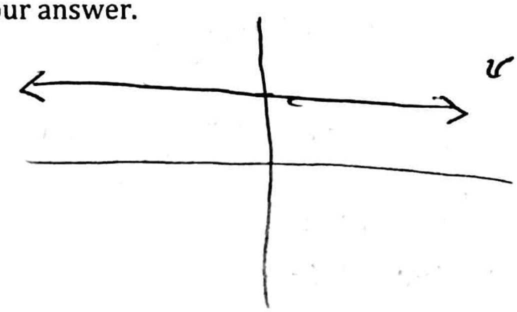
1. A graph of the function g is shown below. Give a graph of g '. Explain how you got your graph.



(1) find whre $f(x)$ has horizontal tangent line

(2) See where $f(x)$ is Increasng and decesng.

2. Explain why $f'(x) = 0$ for $f(x) = c$, where c is a constant. Use a graph to support your answer.



$$\frac{\text{rise}}{\text{run}} = \frac{0}{\text{run}} = 0$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h} = 0.$$

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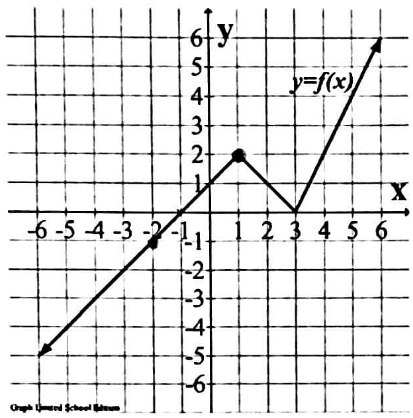
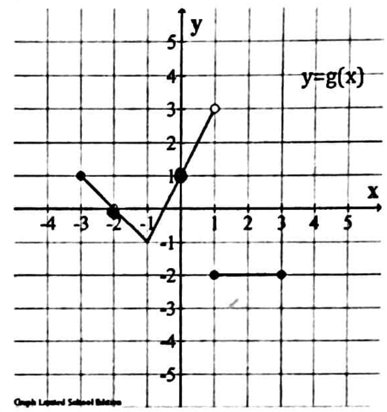
3. The graphs of $f(x)$ and $g(x)$ are given below. Use the graphs in order to answer the following questions.





a) Let $h(x) = f(x)\cdot g(x)$. Find $h'(0)$ and $h'(-2)$

$h'(x) = f'(x)g(x) + f(x)g'(x)$ from graph ↓

$h'(0) = f'(0)g(0) + f(0)g'(0) = 1\cdot 1 + 1\cdot 2 = 3.$

$h'(-2) = f'(-2)g(-2) + f(-2)g'(-2) = 1\cdot 0 + -1\cdot -1 = 1$

b) Let $k(x) = \dfrac{f(x)}{g(x)}$. Find $k'(0)$, $k'(-2)$, and $k'(-1)$.

$k'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

$k'(0) = \dfrac{1\cdot 1 - 1\cdot 2}{2^2} = \dfrac{-1}{4}$

$k'(-2) = \dfrac{1.0 - (-1)(-1)}{(-1)^2} = 0$

$k'(-1) = \dfrac{-1}{1} = -1$

c) Let $w(x) = f(x) - g(x)$. Find $w'(0)$ and $w'(2)$.

$w'(x) = f'(x) - g'(x)$

$w'(0) = 1 - 2 = -1$

$w'(2) = -1 - 0 = -1$

d) $p(x) = 7\cdot f(x) + 64$. Find $p'(-5)$ and $p'(2)$

$p'(x) = 7f'(x)$

$p'(-5) = 7$

$p'(2) = -7$

4. At what point along the curve $y = e^x$ is the tangent line parallel to the line $x - y = 4$? What is the equation of the tangent line to the curve at this point? Graph both the curve $y = e^x$ and this tangent line. Justify your response.

If $e^x \parallel y = x - 4$ then their slopes are equal at that point.

Hence, find when $\frac{d}{dx}(e^x) = 1$

$\frac{d}{dx}(e^x) = e^x = 1$

when $x = 0$.

then $y = 1\cdot x + b$

$1 = 1\cdot(0) + b$

$b = 1$

$\boxed{y = x + 1}$

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5. For the function f(x)=x²eˣ, give all values *c* such that at the point (*c*, f(*c*)) there is a horizontal tangent line. Be sure to explain your steps in finding a solution.

$f'(x) = 2xe^x + x^2e^x$ by product rule

set $f'(x)=0$

$2xe^x + x^2e^x = 0$

$2x + x^2 = 0$ $(e^x \neq 0)$

$x = 0, -2$

$f(0) = 0.$ $f(-2) = 4e^{-2}$

So. $(0,0)$ & $(-2, 4e^{-2})$

6. Find the equation for tangent line to the curve $y = x^2 + 2$ that passes through the point $(6,2)$.

if line is tangent, has same slope.

$\frac{dy}{dx}\Big|_6 = 2x\Big|_6 = 12$

$y = mx + b$

$y = 12x + b$

$2 = 12(6) + b$

$b = 2 - 72 = -70$ $\Rightarrow$ $y = 12x - 70$

7. Compute g '(1), where g(x)=$\frac{x + f(x)}{x - f(x)}$, f(1)=4, and f '(1)=2

$$g'(x) = \frac{(1+f'(x))(x-f(x)) - (x+f(x))(1-f'(x))}{(x-f(x))^2}$$

$$g'(1) = \frac{(1+f'(1))(1-f(1)) - (1+f(1))(1-f'(1))}{(1-f(1))^2}$$

$$= \frac{(3)(-3) - (5)(-1)}{9}$$

$$= \frac{-9+5}{9} = -4/9$$

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8. Assume that f(x) is a differentiable function and that the values of f(x) and its derivative at the points x=0, 1, and 2 are given by:

f(0)=3, f(1)=5, f(2)=-2, and f '(0)=-1, f '(1)=0, f '(2)=3

Let $g(x)=x^2 - 3x \cancel{+} 2$. For each function below calculate the derivative at the given point.

$g'(x) = 2x-3$

a. $\dfrac{f(x)}{g(x)}$, x=0 b. f(x)g(x), x=1 c. $\dfrac{f(x) \times e^x}{g(x)}$, x=2

a) $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

at x=0 $\dfrac{f'(0)g(0) - f(0)g'(0)}{g(0)^2} = \dfrac{(-1)(2)-(3)(-3)}{(2)^2} = \dfrac{7}{4}$

b) $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

at x=1 $f'(1)g(1) + f(1)g'(1) = 0 \cdot 0 + 5 \cdot -1$
$= -5$

c) $\left(\dfrac{f(x)e^x}{g(x)}\right)' = \left(\dfrac{f(x)}{g(x)}\right)' e^x + \left(\dfrac{f(x)}{g(x)}\right) e^x$

at x=2 $\dfrac{7}{4}e^2 + \dfrac{f(2)}{g(2)} e^2$

$= \dfrac{7}{4}e^2 + \dfrac{-2}{0} e^2$

does not exist!