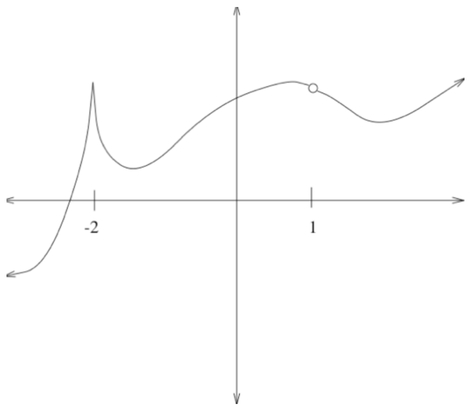
Week 3 Activity – Section 2.6 Continuity and Section 3.1 Definition of the Derivative

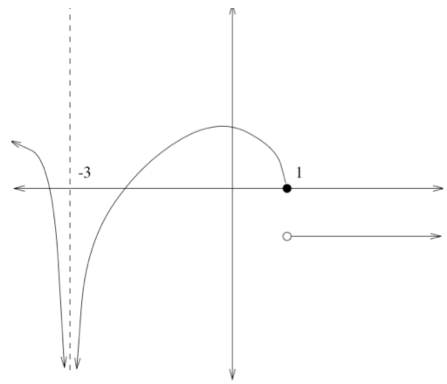
Section 2.6 Continuity

1. In each of the following graphs of functions determine all points of discontinuity and for each point: explain, using the definition of continuity in problem #4 below, why the function is discontinuous at that point. State the intervals on which the function is continuous.

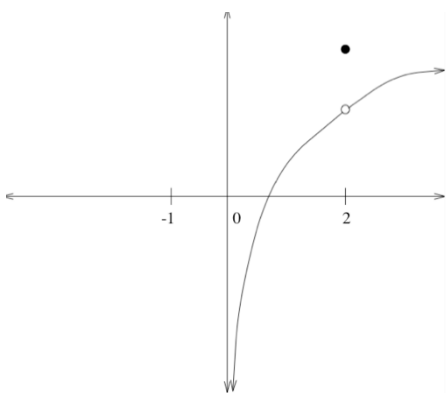
a.



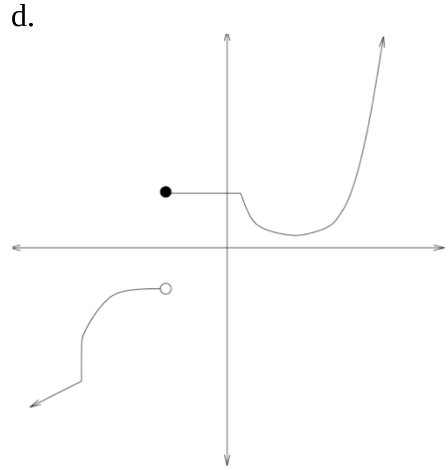
b.



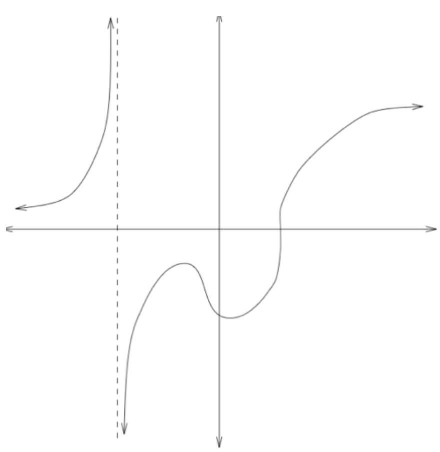
c.



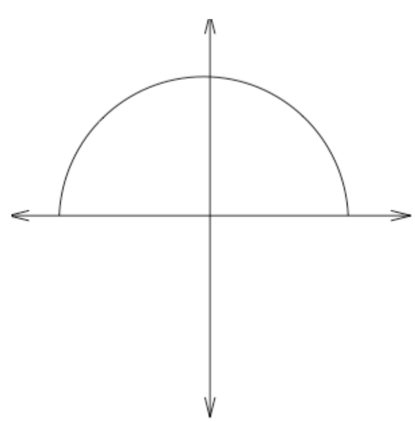
d.



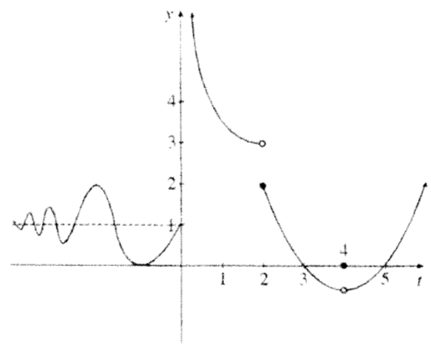
e.



f.



2. Consider the graph of $f(t)$ which is given below.



a. What is $\lim_{t\to 0^+} f(t)$? $\lim_{t\to 0^-} f(t)$? $\lim_{t\to 2^-} f(t)$? $\lim_{t\to 2^+} f(t)$? $\lim_{t\to -\infty} f(t)$?

b. For what values of t is there a vertical asymptote (justify your answer)?

c. Give any horizontal asymptotes for $f(t)$ and justify your answer.

d. For what values of t is $f(t)$ discontinuous? Explain your answer using the definition of continuity (given in problem #4 below).

3. Find examples of functions $\boldsymbol{f}$ and $\boldsymbol{g}$ satisfying the following:

a. $\lim_{x\to a} g(x) = 0$ but $\frac{f(x)}{g(x)}$ does not have a vertical asymptote at x=$\boldsymbol{a}$

b. $\lim_{x\to a} f(x) = 0$ but $\frac{f(x)}{g(x)}$ does have a vertical asymptote at x=$\boldsymbol{a}$

c. $\lim_{x\to a} g(x) \neq 0$ but $\frac{f(x)}{g(x)}$ does have a vertical asymptote at x=$\boldsymbol{a}$

d. $\lim_{x\to a} f(x)$ exists and is not zero but $\lim_{x\to a} g(x)=0$. Does the graph of $\frac{f(x)}{g(x)}$ have a vertical asymptote at x=$\boldsymbol{a}$? Be sure to justify your answer.

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4. A function f(x) is defined to be <u>continuous</u> at the point ***a*** if
   (A) f(***a***) is defined
   (B) $\lim_{x \to a}$ f(x) exists
   (C) $\lim_{x \to a}$ f(x) = f(***a***)

   For each of the following conditions, sketch a graph of a <u>discontinuous</u> function that satisfies the conditions.

   (a) condition (A) holds, but condition (B) does not
   (b) condition (B) holds, but condition (A) does not
   (c) conditions (A) and (B) hold, but condition (C) does not

5. Graph the following function:

$$f(x) = \begin{cases} x^2 + 5, & x < -1 \\ 6x, & x = -1 \\ 27x + 33, & x > -1 \end{cases}$$

   Determine whether f(x) is continuous at x = -1. Fully justify your answer using the definition given in problem #4.

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6. State whether the following functions are continuous or discontinuous at the point ***a*** and explain **<u>why</u>** using the definition of continuity.

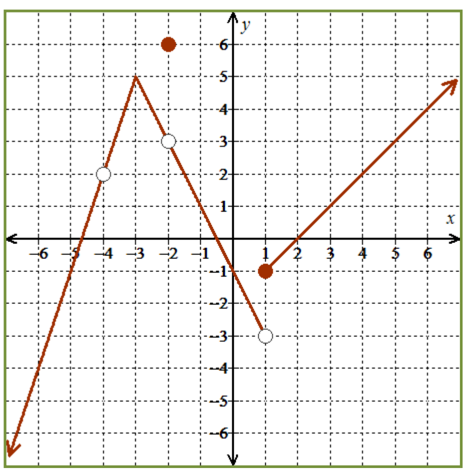
a. $f(x)=x^2+\sqrt{7-x}$ a=4

b. $f(x)=\ln(x-4)$ a=4

7. Find all points of discontinuity for $f(x)=\dfrac{x^2-2x}{x^2+x-6}$ and sketch a graph of the function. State the intervals on which the function is continuous.

8. Determine the intervals on which the following functions are continuous. For each *x* value at which the function is discontinuous, state which of the three continuity criteria fail to be true.

a)



Function: f

b) $f(x)=\dfrac{2x^4-3x^2+2}{x^2+x-6}$

c) $h(x)=\dfrac{x-1}{x^2-1}$

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9. We know the Intermediate Value Theorem:

If $f(x)$ is continuous on $[a,b]$ and there exists 'L' such that $f(a)<L<f(b)$, then there exists 'c' in the open interval (a, b) such that $f(c)=L$.

a. Justify that if $f(x)$ is continuous on $[a, b]$, $f(a) < 0$ and $f(b) > 0$ (or $f(a) > 0$ and $f(b) < 0$) then there exists a 'c' in (a, b) such that $f(c) = 0$.

b. Use the Intermediate Value theorem to show that the equations have a solution on the given interval. Use a graphing calculator for this problem.

i. $2x^3 + x - 2 = 0; (-1, 1)$

ii. $x + e^x = 0; (-1, 0)$